(Re)Dice* Packages for Computer Experiments

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representing all (Re)Dice* package builders:

M. Binois, C. Chevalier, G. Damblin, D. Dupuy, J. Franco,

D. Ginsbourger, C. Helbert, B. Iooss, V. Picheny, Y. Richet

and all other contributors linked to the (Re)Dice projects

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Special acknowledgements

Some R package contributors also contributed to this talk:

- Mickaël Binois (1D EGO slides)
- David Ginsbourger (3D EGO slides)
- Yann Richet

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Outline

- Computer Experiments and the (Re)Dice consortiums
- Designs for Computer Experiments
- Kriging Models and Gaussian Processes
- Kriging-based Optimization
- Playing with Kernels
- Practical Considerations

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Part I

Computer experiments and (Re)Dice consortiums

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What are Computer Experiments?



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Who, where?

- Automotive
- Aerospace
- Oil
- Nuclear industry
- Hydrology, climate analysis
- Carbon sequestration

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See e.g. the books by Fang et al [FLS06] or by Santner et al. [SWN03]

What for? (unclosed list...)

- Design of Computer Experiments
 - \rightarrow choose inputs levels in order to best approximate the output
- Interpolation/approximation
- Global Optimization
- Sensitivity Analysis

 \rightarrow quantify the global effects of inputs on one output

Inversion

 \rightarrow find regions where f_{sim} remains below some fixed threshold

Risk Assessment

 \rightarrow e.g. estimate a quantile of an output for random inputs

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How?

Metamodels come from old and new branches of statistics

- Polynomial models
 - \rightarrow response surface
- Gaussian Processes and Kriging

 \rightarrow widely used in spatial statistics or spatio-temporal statistics

• Models from Statistical Learning

 \rightarrow SVM, RBF, Neural Networks, GAM, Mars, PolyMars, ...

Polynomial Chaos

Dice and ReDice consortiums

Principle: get together with partners willing to share efforts on computer experiments

- Open problems
- Case studies
- Ideas, tips
- Code
- Training
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Dice Consortium (2006-2009)



dice.emse.fr

- 5 industrials (EDF, IRSN, ONERA, Renault, TOTAL)
- 4 academics (<u>EMSE</u>, Univ. Aix-Marseille, Grenoble and Orsay)
- 3 PhD thesis funded (J. Franco, D. Ginsbourger, V. Picheny)

Outputs

- +30 internal reports, 8 publications
- 4 R packages released on CRAN
 - \rightarrow DiceDesign, DiceEval, DiceKriging, DiceOptim

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Dice Consortium (2006-2009)



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ReDice Consortium (2011-2015)



www.rediceproject.org

- 5 industrials (EDF, CEA, IFPEN, IRSN, Renault)
- 5 academics (EMSE, Univ. <u>Bern</u>, Grenoble, Toulouse, Nice)
- 3 PhD thesis funded (M. Binois, C. Chevalier, F. Zertuche)

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Differences with Dice

- Publication is enhanced by light property constraints
 - $\rightarrow \emptyset$ for methodological articles, 6 month delay for R packages
- Training sessions (maths + software)

ReDice Consortium (2011-2015)



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Why R?

Existing software in computer experiments: Written in C, Python, popular matrix languages, ...



- was immediately well-accepted by the Consortiums
- Lingua Franca of statistical computing.
- Works on major platforms.
- Open-source and easy to extend.
- Can be used in many modes or fashions.
- Huge choice of updated CRAN packages.
- \rightarrow write new packages and/or enhance existing ones!

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How new R packages are used

Industrials

- To solve real-life problems with new methods.
- Often used in a complex computing environment, e.g. Java.
 - \rightarrow *Prométhée* at IRSN, see later

Academics

- For methodological developments.
- Case studies, toy examples, benchmarks.
- Construction efforts are often valorized by publications.

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Part II

Designs for Computer Experiments

Design specificities for computer experiments

Two main constraints and their consequences.

 For deterministic simulators, running twice the simulator at the same location gives the same result

 \rightarrow Avoid replications

A simulator generally models a complex phenomenon with strongly non-linear behavior

 \rightarrow Fill the space, in order to avoid missing an area

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Additional constraint:

The aforementioned principles should apply in lower subspaces, since the *true* dimensionality can be lower.

Example: $f_{sim}(x_1, x_2) = g(x_1)$

- \rightarrow It is useless to run the simulator with same x_1 locations!
- \rightarrow The x_1 locations should fill the space

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A potentially bad design: How to waste 2/3 of runs!

Only 3 points among 9 are useful if f_{sim} depends only on x_1 (or x_2).



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A potentially bad design: How to waste 2/3 of runs!

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Better designs: Latin hypercubes R> X <- lhsDesign(n = 9, dimension = 2, seed = 3, randomized = FALSE)\$design



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Better designs: Latin hypercubes R> X <- lhsDesign(n = 9, dimension = 2, seed = 3, randomized = FALSE)\$design



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Good(?) designs: space-filling Latin hypercubes
R> X0 <- lhsDesign(n = 9, dimension = 2)\$design
R> X <- maximinESE_LHS(X0, it=2)\$design</pre>



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Evaluation of designs with the radial scanning statistic

The radial scanning stat. tests if the projections onto a straight line are compatible with uniformity distribution in the whole domain.

Example with a 80-point low discrepancy (Sobol) sequence, obtained with package **randtoolbox**.



Top: Radial scanning statistic for the Sobol sequence, indicating potential problems if $f_{sim}(\mathbf{x}) = g(x_2 - x_7)$. Bottom: The problem disappears with scrambling.



DiceDesign features

Onstruction of space-filling designs

- Maximin Latin hypercubes
- Other designs: Strauss, maximum entropy, WSP, ...

 \rightarrow With a special care on optimization procedures

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DiceDesign features

Construction of space-filling designs

- Maximin Latin hypercubes
- Other designs: Strauss, maximum entropy, WSP, ...

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- e Evaluation of designs
 - Computation of criteria: distances, discrepancies
 - \rightarrow Discrepancies evaluate the distance to uniformity
 - Graphical tool: radial scanning statistic

 \rightarrow Detect (oblique) alignments, that may induce a lost of information

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Part III

Kriging Models and Gaussian Processes

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Kriging models and Gaussian processes for deterministic simulators

A Kriging model is a Gaussian process (GP) of the form:

$$Y(\mathbf{x}) = \underbrace{\beta_0 + \beta_1 g_1(\mathbf{x}) + \dots + \beta_{p-1} g_{p-1}(\mathbf{x})}_{m(\mathbf{x})} + Z(\mathbf{x})$$

where:

- **x** is the *location*, typically a vector of length d
- $m(\mathbf{x})$ is a deterministic *trend*, linear comb. of given g_i 's
- $Z \sim GP(0, k)$ where k is the covariance kernel,

$$k(\mathbf{x}, \mathbf{x}'; \Theta) = \operatorname{cov}(Z(\mathbf{x}), Z(\mathbf{x}'))$$

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 \rightarrow A deterministic simulator is viewed as a particular path of Y



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Figure: Simulated paths of a Kriging model (Matérn kernel)

Link to geostatistics

The Kriging model originates from geostatistics and the work of D. Krige [Kri51], followed by G. Matheron [Mat63] and many others. The problem dimension d is then 2 or 3. Examples:

- x is the 3D coordinate in the subsoil, Y(x) is the amount of gold per volume
- **x** is the 2D location in France, $Y(\mathbf{x})$ the rainfall

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Link to time series

• Linear model with AR(1) errors:

$$Y(t) = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_{p-1} x_{p-1,t} + Z(t), \quad t = 1, 2, \dots$$

where $Z(t) = \phi Z(t-1) + \varepsilon(t)$, with $\varepsilon(t)$ is a Gaussian white noise.

 \rightarrow Here, k is an exponential kernel $k(t, t') = \sigma^2 \exp(-|t - t'|/\theta)$

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Kriging: An interpolator with prediction bounds!

Prediction with Kriging model is obtained by *Gaussian vector conditioning*. Given:

- A set of locations $\mathbf{X} = \mathbf{x}^1, \dots, \mathbf{x}^n$,
- A set of observations $\mathbf{y} = y^1, \dots, y^n$, with $y^i = (f_{sim}(x^i))$.

By conditioning on $Y(\mathbf{x}^1) = y^1, \dots, Y(\mathbf{x}^n) = y^n$, we obtain a GP:

• Whose paths interpolate the observations

 \rightarrow The mean – *Kriging mean* – is an interpolator

• Whose kernel does not depend on the observations

 \rightarrow The conditional variance – Kriging variance – does not depend on y^1,\ldots,y^n







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Kernels

GP can be customized by choosing appropriate covariance kernels,

$$k(\mathbf{x}, \mathbf{x}') = \operatorname{cov}(Z(\mathbf{x}), Z(\mathbf{x}'))$$

Below some operations to construct new kernels from old:

- Product: $k_1(\mathbf{x}, \mathbf{x}') \times k_2(\mathbf{x}, \mathbf{x}')$
- Tensor product: $k_1(\mathbf{x}_1, \mathbf{x}'_1) \times k_2(\mathbf{x}_2, \mathbf{x}'_2)$
- Sum, tensor sum
- Change of scale: $k(g(\mathbf{x}), g(\mathbf{x}'))$
- ...

 \rightarrow Usual *d*-dim. kernels are tensor product of 1D kernels

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Extension to noisy observations

In various situations (ex: Neutronics), the simulator is stochastic.

• The Kriging model has to be adapted:

$$\zeta_i = Y(\mathbf{x}^i) + \varepsilon_i, \qquad i = 1, \dots, n$$

where Y is a GP and ε_i are ind. Gaussian r.v. $N(0, \sigma_i^2)$.

 \rightarrow We can have several observations at the same location

The aim is to predict Y(x) conditionaly on ζ₁,..., ζ_n
 → A filtering problem, also solved by Gaussian conditioning,

which only slightly modifies the Kriging formulas

• A.k.a. Gaussian Process Regression.



Figure: The noisy observations (stars) are obtained from the true function $Y(\mathbf{x})$ (dotted line) by adding an heterogeneous noise (vertical bars). Confidence bounds in shaded grey. The Kriging mean is no more an interpolator.

Parameter estimation

The trend and covariance parameters are often unknown.

• Estimation can be done numerically by MLE or by Cross-Validation (CV)

 $\rightarrow\,$ The analytical gradient of the criteria is supplied to the optimizers, enhancing their efficiency

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The complexity for computing the criteria is O(n³)
 → A problem when n is large, but often not in our context of time-consuming simulators.

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A 6-dimensional (toy) example

Let us choose the Hartman function, transf. by $y \mapsto -\log(-y)$.

Choose a 80-point maximin LHS design. R> X0 <- lhsDesign(n = 80, dimension = 6)\$design R> X <- maximinSA_LHS(X0)\$design</p>



\bigcirc Fit a Kriging model to the observations y

```
R> m <- km(design = data.frame(X), response = y,
+ control = list(trace=F))
```

 \rightarrow we use the default formula

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R> m

```
Call:
km(design = data.frame(X), response = y, control = list(trace = F))
Trend coeff.:
             Estimate
 (Intercept) 4.4372
Covar. type : matern5_2
Covar, coeff.:
             Estimate
  theta(X1) 0.6640
  theta(X2) 0.9639
  theta(X3) 1.9630
  theta(X4) 0.7540
  theta(X5) 0.6893
  theta(X6)
               0.5600
```

Variance estimate: 4.812747







Leave-one-out





Theoretical Quantiles

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 Predict at new locations, here a 250-point random Latin hypercube, with method predict. Compare with true values.

R> y.pred <- predict(m, newdata = X.test, type = "UK")</pre>



DiceKriging features

- For both deterministic or noisy observations
- Accepts a general linear trend

 \rightarrow With the formula mechanism

• Various classes of kernels already implemented

 \rightarrow Possibility to add one's kernel (no parameter estimation)

- Parameter optimization: MLE or CV, with known trend or known covariance, classic or genetic optimization, choice of control parameters, ...
- Methods: simulate, predict, plot

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DiceKriging: a shiny demo

Yann Richet has written a shiny animation showing a km fit, see it at http://glimmer.rstudio.com/richetyann/DiceKriging.



Part IV

Kriging-based Optimization

$Metamodel-based\ optimization$

The aim is to minimize f_{sim} , with the help of a Kriging model Y.

- Wrong way: To minimize the Kriging mean.
 → Highly depends on the quality of the first interpolation!
- Right way: To use both the Kriging mean & variance
 → Results in efficient sequential strategies

Expected Improvement EI

• Improvement: What is below the current minimum f_{min}

$$I(\mathbf{x}) := \max(f_{\min} - Y(\mathbf{x}), 0)$$

Expected Improvement: Expectation of the r.v. *I*(**x**), conditionaly on the observations **y** at **X** = (**x**¹,..., **x**ⁿ):

$$\mathsf{EI}(\mathbf{x}) := \mathbb{E}\left[I(\mathbf{x})|Y(\mathbf{x}^1) = y^1, \dots, Y(\mathbf{x}^n) = y^n\right]$$

 \rightarrow El(x) has an analytical expression depending on the Kriging mean and Kriging variance

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EGO: EI-based sequential optimization strategy

Start with an initial Kriging model. Then repeat until a stopping criterion is reached:

- Find x* which maximizes El(x)
- ② Evaluate the simulator at x*
- $\textbf{0} \quad \text{Update the learning set: } \textbf{X} \leftarrow \textbf{X} \cup \{\textbf{x}^{\star}\} \text{ , } \textbf{y} \leftarrow \textbf{y} \cup \{f_{\min}(\textbf{x}^{\star})\}$
- (Possibly) reestimate the Kriging model

 \rightarrow "Efficient Global Optimization" (EGO) algorithm of Jones et al [JSW98]

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Left: Kriging model surface. Right: El surface.



Left: Kriging model surface. Right: El surface.





Left: Kriging model surface. Right: El surface.



Left: Kriging model surface. Right: El surface.



Adaptation to a parallel setting

Aim: To give a *batch* of points at each step

- q-EI: Generalizes EI for a batch of *q* points
 → A batch gives an improvement if at least one is better
- Liar strategies: Apply q-times the 1-step El strategy without
 - evaluating the simulator

 \rightarrow Provide a 'reasonable' value of $\mathit{f}_{\rm sim}(\bm{x}^{\star}),$ typically the current minimun value

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batch 1





Example: Usage of **DiceOptim** at IRSN



promethee. irsn.org

Prométhée is a software workbench created by IRSN. It provides a Graphical User Interface for distributed automated parametric computation on cluster, workstations, desktop, ... It works with several simulators: MCNP(X), Moret, Apollo, ...

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Prométhée is based on Java and RServe by S. Urbanek [Urb13].

- ඹ plays a crucial role in several tasks.
 - Parameterization of input files.
 - Metamodels, optimization, sensitivity analysis, ...
 - Scripting language.
 - Output management (web).

(Re)Dice packages are widely used.

Example: Usage of **DiceOptim** at IRSN



Screenshot of Richet et al [RGRD10]. The ask and tell mechanism is an original idea arising from package **sensitivity** [PIJ13].

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DiceOptim features

Kriging-based optimization of deterministic simulators

- El algorithm (EGO)
- Parallel El algorithm: q-points El, CL heuristics

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DiceOptim features

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- Ø Kriging-based optimization for noisy observations
 - Noisy El-like criteria: Expected Quantile Improvement (EQI), Augmented El (AEI), Approximate Knowledge Gradient (AKG)
 - Corresponding sequential optimization strategies

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 - Corresponding sequential optimization strategies

 \rightarrow For most criteria (EI, EQI, AKG), the analytical gradient is supplied, enhancing the algorithms efficiency

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Kernels

Part V

Playing with Kernels

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Special kernels

 $\ensuremath{\text{DiceKriging}}$ allows the user to write her/his own kernel from a simple R function.

This function can be "inlined" using the inline package [SMS+13].

Example: kernel with an *invariance property*, e.g. symmetry. Specificity of computer experiments; usually not a concern in spatial stat. or geostatistics. Invariant kernels has been a theme of research in Dice and ReDice consortiums, see Ginsbourger et al. [GBRC12].

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Special kernels

$$4 k(x_1, x_2) = k^*(x_1, x_2) + k^*(s.x_1, x_2) + k^*(x_1, s.x_2) + k^*(s.x_1, s.x_2)$$

where s is the symmetry $x \mapsto 1 - x$ and k^* is a kernel. The sample paths become symmetrical.

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Special kernels



Figure: We can use predict, simulate, etc. symmetry automatically results. Left: prediction from 5 points. Right: conditional simulations

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Special kernels

This generalizes to kernels invariant by a finite group ${\mathcal{G}}$ of transformations

$$k(\mathbf{x}_1, \mathbf{x}_2) \propto \sum_{s_1, s_2 \in \mathcal{G}} k^{\star}(s_1.\mathbf{x}_2, s_2.\mathbf{x}_2)$$

the dimension d and the kernel k^* being arbitrary.

 $\rightarrow \mathcal{G}$ was a group of 2 transformations id and \emph{s} in previous example.

 \rightarrow We can play with two axial symmetries in dimension d=2. Then ${\cal G}$ has 4 elements.

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KERNELS

Special kernels



GRF Simulation relying on an invariant kernel

Simulation from a kernel for dimension d = 2 with two axial symmetries.

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Special kernels

User defined kernels have been made more performant and flexible in package **DKlab** (forthcoming).

- Parameter estimation by Maximum Likelihood.
- Optimization of performance through .Call.

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Part VI

Practical Considerations

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Package development

A few hints arising from our experience about R packages development within the consortiums(s): technical and non-technical considerations.

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Take time to learn R

- Some key R objects must be perfectly known or understood: \rightarrow data.frame \neq matrix, factor, ...
- A R function or *closure* is an amazingly powerful thing!
 → but some practice is needed to make good use of it: *dots*, missing formals, scoping, ...
- Read again and again the manual Writing R Extensions.
- Take time to study the code of good existing packages.

 \rightarrow Packages written by R core members are of great help

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Make your life easier

- Dramatic productivity gains can be reached by using RStudio for package development. This is especially true for new developers and PhD students.
- ... but some still love emacs/ESS and command lines R CMD build
- Packages such as roxygen2, testthat are of great value.
- Use a Version Control System.
- It is a real strength to co-work on a variety of platforms, text editors, graphical devices, ...

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Package design

Kriging metamodels are not unlike statistical models...

• The data/formula interface is very flexible to create meta-models.

• But must sometimes be completed.

 \rightarrow GP models have predictors and inputs. The order of the inputs is important.

• Implementing classical methods make a package easier to use: summary, coef, predict, simulate, ...

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Methods

- Methods (S3, S4 or R5) enhance code reliability.
- Methods can allow users to extend our (Re)Dice packages *from outside*.

 \rightarrow write a new kernel class and a few methods as in nlme, among which coef and 'coef<-'

• But writing methods requires a clear vision of the final code.

 \rightarrow using S3 temporarily can be a solution

• When you find duplicated code, consider writing a method.

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Documentation

- Writing good documentation is difficult and time-consuming
 → documenting S4 methods often generates headaches
 Write vignettes or reproducible research documents!
 - $\rightarrow~$ Who reads the pdf version of a package's manual?

Compiled code

- Writing compiled code is necessary for some computational CE tasks: *kriging*, building complex designs, ...
 - \rightarrow C, C++ or Fortran
- Using .C in a package is fairly simple and efficient.

 $\rightarrow\,$ however, with much object duplication

- Very few people enjoy writing code with .Call.
 - $\rightarrow~$ the use of macros can be intimidating
- Consider using RCpp! [EF11]

 $\rightarrow \,$ there is certainly a good C/C++ programmer in an office next to your's.

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Project management

- It is quite difficult for one person to write a package of CRAN (or higher) quality. This can be time-consuming.
- It is necessary to have feedback during the package development.
- Proving theorems and writing good R code is a real challenge for applied math PhD students!

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Authorship

• Dice and even more ReDice consortiums are very well suited to support and encourage R package development

 \rightarrow at the present time, the CRAN policy may discourage some companies: maintainer and author(s) must be *persons*, which may be a problem.

 Urge academics/researchers on properly citing R CRAN packages (and their authors) as they do with research articles.

Thank you!

Merci à Kurt Hornik, Uwe Ligges et Brian Ripley, R core members

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