École Nationale Vétérinaire, Agroalimentaire et de l'Alimentation Nantes Atlantique

Unité de Sensométrie et Chimiométrie

"BLOCKBERRY" : AN EXPERIMENTAL R PACKAGE FOR BLOCK MATRICES

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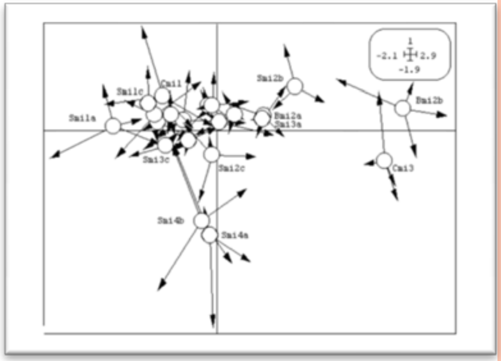
MAIN AIMS

- Introduce an algebraic framework which extends algebra from matrix to block matrix.

- Provide an operational solution for fast prototyping multiblock algorithms (R package “BlockBerry”)



- Computational core to design packages for analyzing multiblock data

ISSUES AND CONTRIBUTIONS (1)

- Clarify definitions and vocabularies for block matrices and block tensors.
  - What is a block matrix and what it is not?

- Introduce the block dimension concept for block matrices and block tensors (as an extension of the usual dimension of matrices).
  - how to take into account the block structure ?

- Storage of block matrices or block tensors on scientific software (like R or Matlab,…).
  - how to handle block matrix or block tensors on scientific software (here R)?

BLOCK DIMENSION (1)

PARTITION OF AN INTEGER

- A partition of an integer *n* in *k* parts is a decomposition of *n* as :

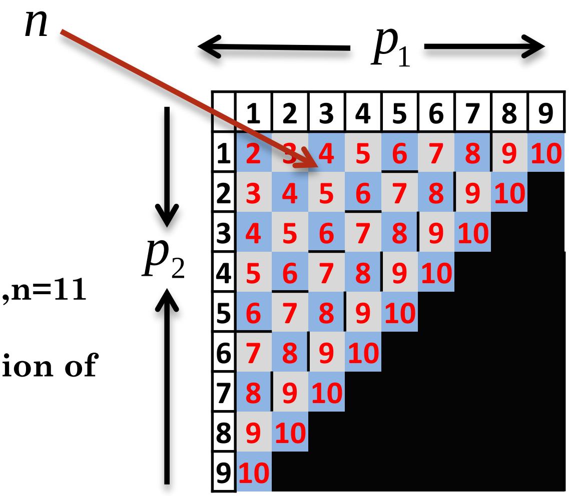
$$n = \sum_{i=1}^{k} p_i \qquad (p_i \neq 0, 1 \leq i \leq k)$$

- The integers $(p_1, p_2, \ldots, p_k)$ are called the parts of the partition
- $\mathbf{P}_{(k)}(n)$ denotes the set of all partitions the integer *n* in *k* parts.

$$\text{Sets}: \mathbf{P}_{(2)}(2), \ldots, \mathbf{P}_{(2)}(10)$$

$$\mathbf{P}_{(2)}(4) = \{(3,1), (2,2), (1,3)\}$$

Sets of partitions of integers n=1, n=2,…,n=11 in k=2 parts.
Each pair of entries (i,j) defines a partition of the elements of the symmetric matrix



	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	
3	4	5	6	7	8	9	10		
4	5	6	7	8	9	10			
5	6	7	8	9	10				
6	7	8	9	10					
7	8	9	10						
8	9	10							
9	10								

BLOCK DIMENSION (2)

PARTITIONS OF A PAIR OF INTEGER

- For a given pair of integer $\mathbf{n}=(n_1,n_2)$,
- The set $\mathbf{P}_{(k_1,k_2)}(n_1,n_2)$ is defined as :

$$\mathbf{P}_{(k_1,k_2)}(n_1,n_2)=\mathbf{P}_{(k_1)}(n_1)\times\mathbf{P}_{(k_2)}(n_2)$$

- $\mathbf{P}_{(k_1,k_2)}(n_1,n_2)$ is called the set of partitions of the pair of integers $\mathbf{n}=(n_1,n_2)$ into (k_1,k_2) parts

Integer vectors	Partitions
(6,4)	((2,2,2),(4))
10	(6,4)
(2,8)	((2),(3,5))
(5,5)	((2,3),(5))

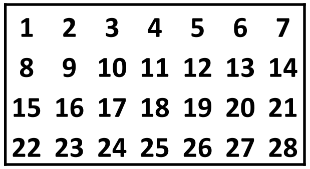
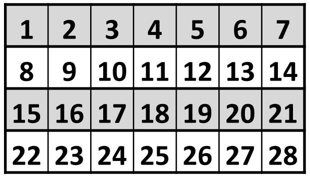
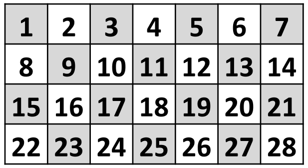
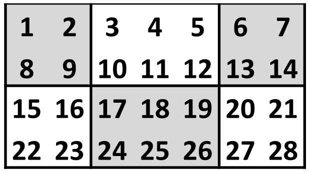
BLOCK DIMENSION (3)

BLOCK MATRIX DEFINITION

- Elements of $\mathbf{M}_{(n_1,n_2)} \times \mathbf{P}_{(k_1,k_2)}(n_1, n_2)$ are called block matrices.
- A block matrix is a pair $\left(\mathbf{X}, P_{(k_1,k_2)}(n_1, n_2)\right)$ where $\mathbf{X}$ is a matrix of dimension (n_1, n_2) with its partition $P_{(k_1,k_2)}(n_1, n_2)$
- $P_{(k_1,k_2)}(n_1, n_2)$ is called the block dimension of X
- Block dimension defines the block structure of X

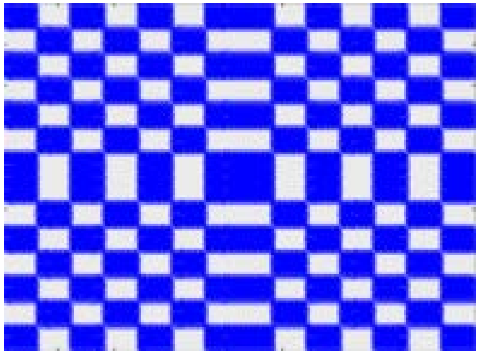
BLOCK DIMENSION (4)

EXAMPLES OF BLOCK MATRICES

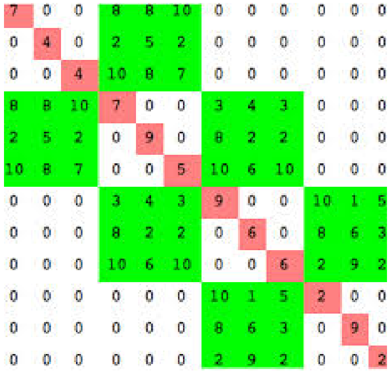
Block matrix	**BlockDimension**	**Block structure**
 $\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ 22 & 23 & 24 & 25 & 26 & 27 & 28 \end{matrix}$	((4),(7))	(1,1)
 $\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ 22 & 23 & 24 & 25 & 26 & 27 & 28 \end{matrix}$	((1,1,1,1),(7))	(4,1)
 $\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ 22 & 23 & 24 & 25 & 26 & 27 & 28 \end{matrix}$	((1,1,1,1),(1,1,1,1,1,1,1))	(4,7)
 $\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ 22 & 23 & 24 & 25 & 26 & 27 & 28 \end{matrix}$	((2,2),(2,3,2))	(2,3)

REMARK

Following our definitions, blocks in block matrices must be rectangular



This is a block matrix



7	0	0	8	8	10	0	0	0	0	0	0
0	4	0	2	5	2	0	0	0	0	0	0
0	0	4	10	8	7	0	0	0	0	0	0
8	8	10	7	0	0	3	4	3	0	0	0
2	5	2	0	9	0	8	2	2	0	0	0
10	8	7	0	0	5	10	6	10	0	0	0
0	0	0	3	4	3	9	0	0	10	1	5
0	0	0	8	2	2	0	6	0	8	6	3
0	0	0	10	6	10	0	0	6	2	9	2
0	0	0	0	0	0	10	1	5	2	0	0
0	0	0	0	0	0	8	6	3	0	9	0
0	0	0	0	0	0	2	9	2	0	0	2

Not a block matrix,
but can be written as finite sum
of block matrices

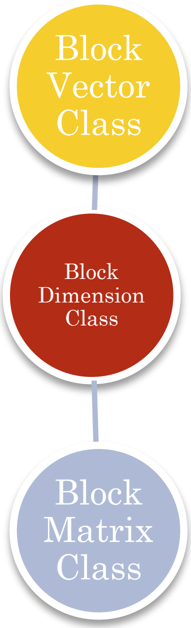
REPRESENTATION OF BLOCK DIMENSION

Integer	Mathematical representation	Attributes of BlockDimension Class	
		Parts	Dims
(4,7)	((1,1,1,1),(7))	(1,1,1,1,7)	(4,1)
	((4),(7))	(4,7)	(1,1)
	((1,1,1,1),(1,1,1,1,1,1,1))	(1,1,1,1,1,1,1,1,1,1,1)	(4,7)
	((4),(1,1,1,1,1,1,1))	(4,1,1,1,1,1,1,1)	(1,7)
	((2 ,2), (2, 3, 2))	(2 ,2,2, 3, 2)	(2,3)

- Representation is the way to store partition of integers (as a pair of integer vectors)

PRELIMINARY

- block-dimension concept is implemented as an S4 class under the name "Blockdimension".
- the block-Vector object as a vector with a block-dimension attribute
- a block-matrix object is a matrix with the attribute block-dimension.



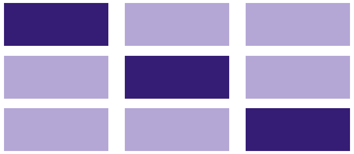
Block Vector Class

Block Dimension Class

Block Matrix Class

BLOCKBERRY

An experimental R package
for Handling block matrices in R.



Installing “blockberry”

https://github.com/gastonstat/blockberry

(not yet in CRAN)

```
# install devtools
install.packages(“devtools”)

# load devtools
library(devtools)

# install blockberry (from github)
install_github(“devtools”,
“gastonstat”)

# load blockberry
load(blockberry)
```

```r
# create a matrix
m = matrix(1:28, 4, 7)
```

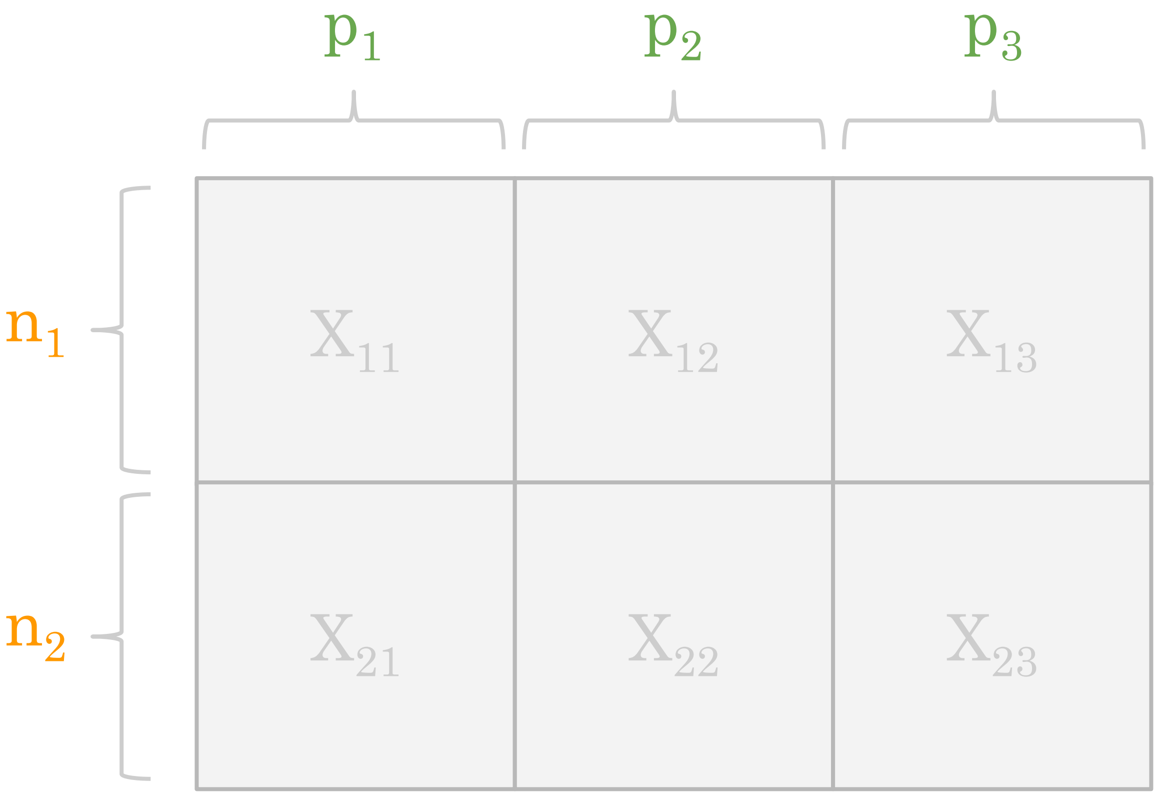
1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

Option 1) Specifying bloc row and block column partitions

X = blockmatrix(X, rowparts = c(n_1, n_2),colparts = c(p_1, p_2, p_3))

Option 2) Specifying block-dimension components (parts, dims)

X = blockmatrix(X, parts = c(n_1, n_2, p_1, p_2, p_3), dims = c(2, 3))



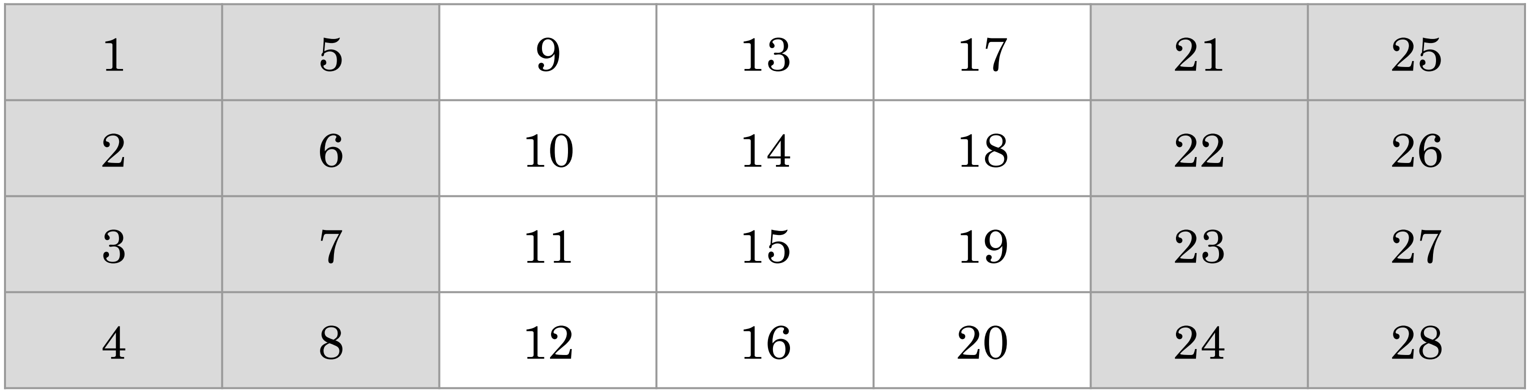
```
# blockmatrix (one block)
bm = blockmatrix(m,
           parts = c(4, 7),
           dims = c(1, 1))
```

1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

```
# blockmatrix (2 column blocks)
bm1 = blockmatrix(m,
           parts = c(4, 4, 3),
           dims = c(1, 2))
```

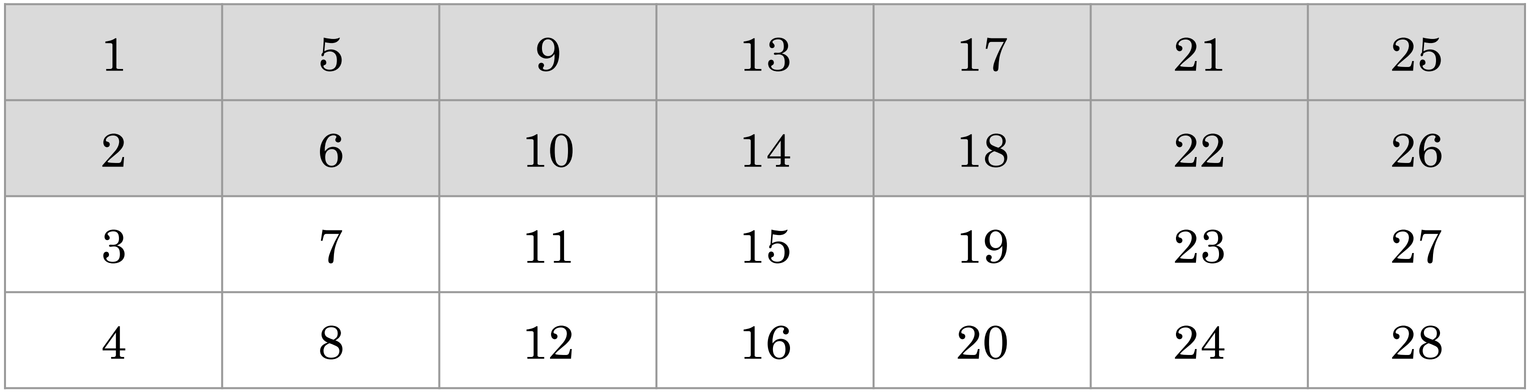
1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

```
# blockmatrix (3 column blocks)
bm2 = blockmatrix(m,
          parts = c(4, 2, 3, 2),
          dims = c(1, 3))
```



1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

```
# blockmatrix (2 row blocks)
bm3 = blockmatrix(m,
             parts = c(2, 2, 7),
             dims = c(2, 1))
```



1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

```
# blockmatrix (4 row blocks)
bm4 = blockmatrix(m,
          parts = c(1, 1, 1, 1, 7),
          dims = c(4, 1))
```

1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

```
# blockmatrix (4 blocks)
bm5 = blockmatrix(m,
       parts = c(2, 2, 3, 4),
       dims = c(2, 2))
```

1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

How to modify blockdimension?

```
blockdim() <- list()
```

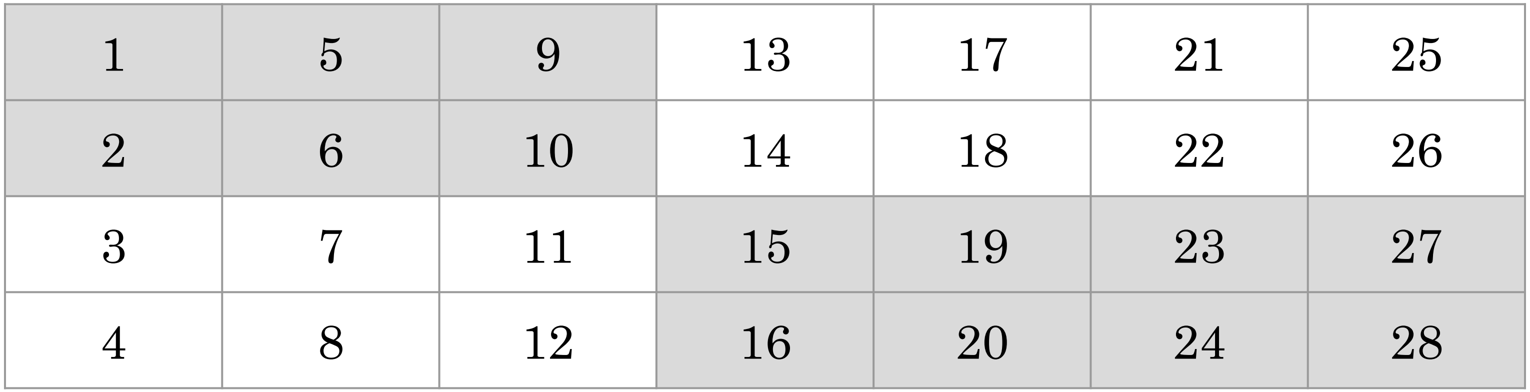
Modifying blockdimension

$$n_1 + n_2 = m_1 + m_2 + m_3$$

$$p_1 + p_2 + p_3 = q_1 + q_2$$

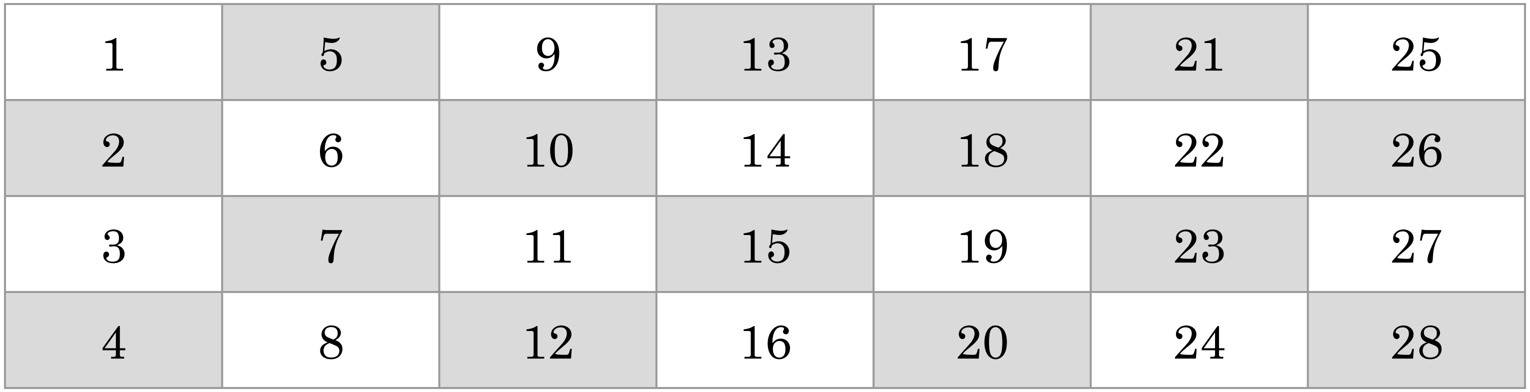
$$\begin{array}{c|c|c|c|}
 & p_1 & p_2 & p_3 \\
\hline
n_1 & X_{11} & X_{12} & X_{13} \\
\hline
n_2 & X_{11} & X_{12} & X_{13} \\
\hline
\end{array}
\Rightarrow
\begin{array}{c|c|c|}
 & q_1 & q_2 \\
\hline
m_1 & X^*_{11} & X^*_{12} \\
\hline
m_2 & X^*_{21} & X^*_{22} \\
\hline
m_3 & X^*_{31} & X^*_{23} \\
\hline
\end{array}$$

```
# blockmatrix (4 blocks)
A = blockmatrix(m,
       parts = c(2, 2, 3, 4),
       dims = c(2, 2))
```



1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

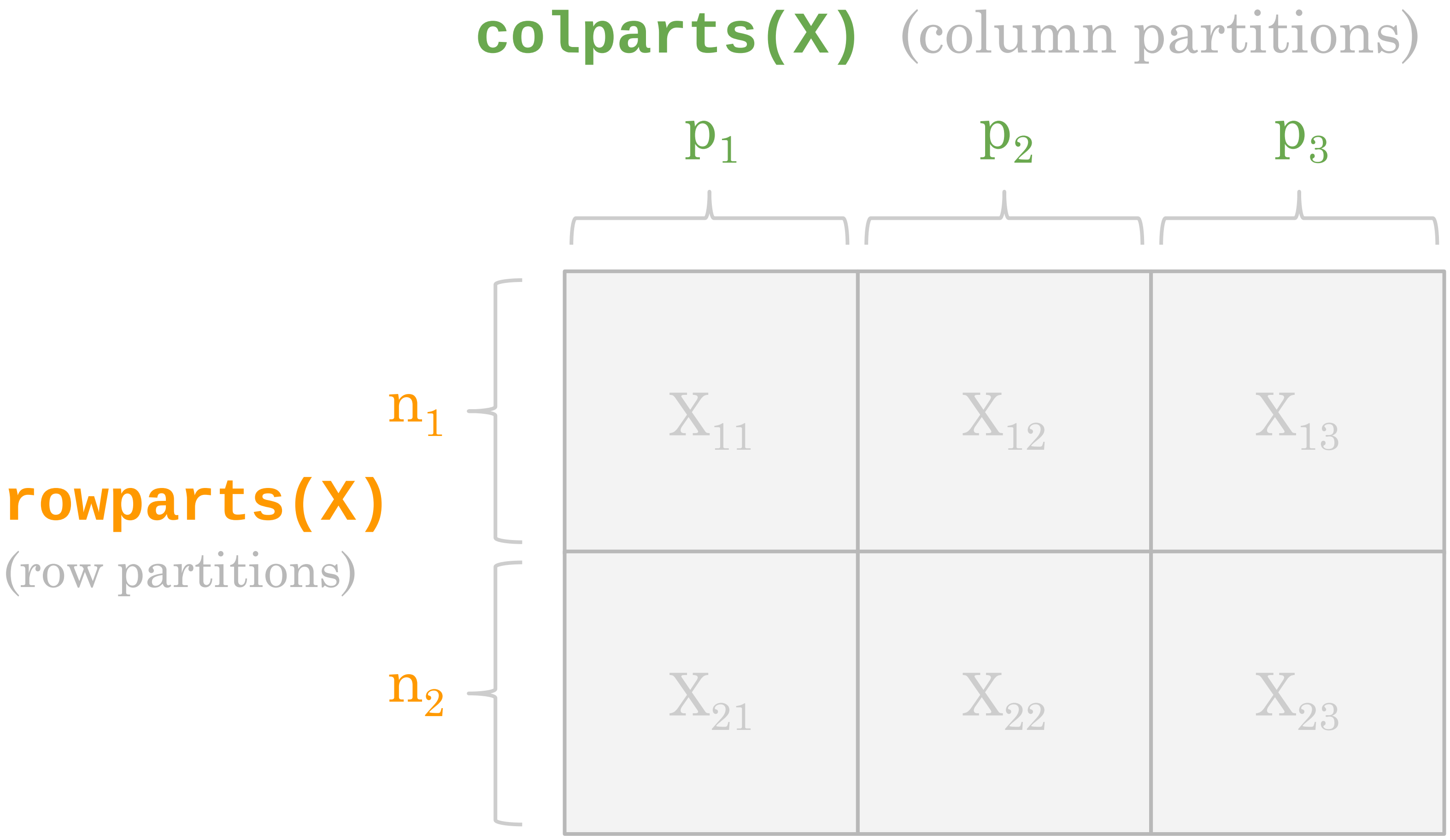
```
# equivalent to bm6
blockdim(A) = list(
     parts = c(rep(1, 4), rep(1, 7)),
     dims = c(4, 7))
```



1	5	9	13	17	21	25
2	6	10	14	18	22	26
3	7	11	15	19	23	27
4	8	12	16	20	24	28

How to inspect blockmatrices?

`blockdim(), parts()`
`dims(), nblocks(),`
`bdim(), brow(), bcol()`
`rowparts(), colparts()`
`describe()`



`colparts(X)` (column partitions)

p_1 p_2 p_3

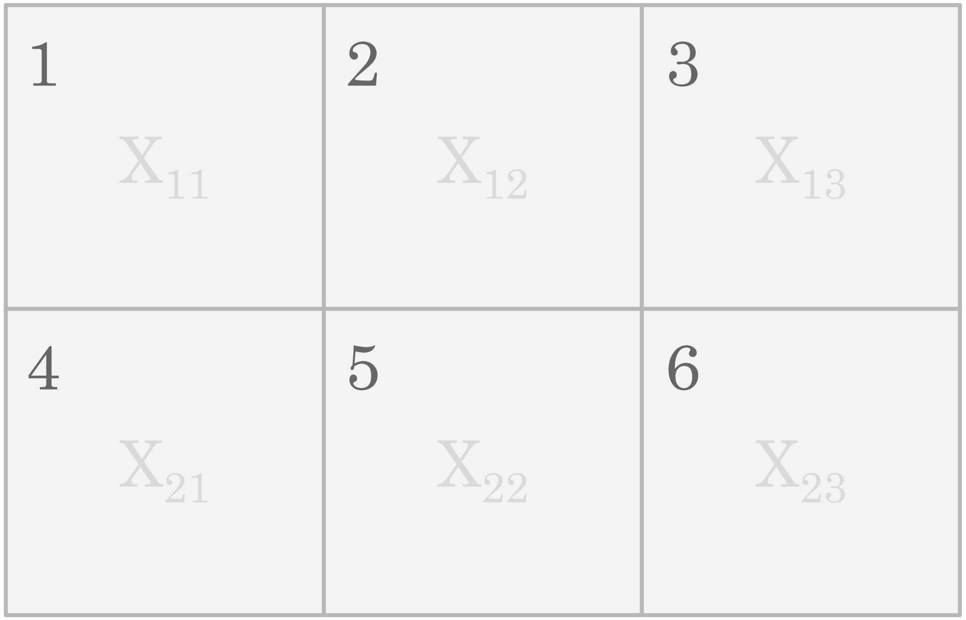
`rowparts(X)` (row partitions)

n_1 n_2

$$\begin{array}{c|ccc} & p_1 & p_2 & p_3 \\ \hline n_1 & X_{11} & X_{12} & X_{13} \\ n_2 & X_{21} & X_{22} & X_{23} \end{array}$$

`nblocks(X) = 6`

total number of blocks



1 X_{11}	2 X_{12}	3 X_{13}
4 X_{21}	5 X_{22}	6 X_{23}

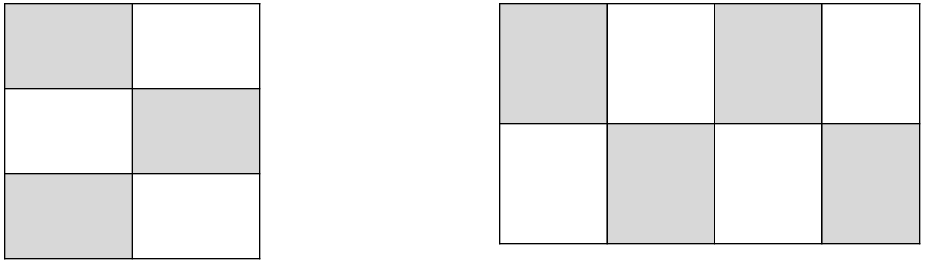
More manipualtions

Function	*Description*	*Function*	*Description*
`blockdim()`	block-dimension	**`nblocks()`**	total # of blocks
`parts()`	partitions	**`get_block()`**	extract specific block
`dims()`	dimensions	**`rowblock()`**	extract row block
`rowparts()`	row partitions	**`colblock()`**	extract column block
`colparts()`	column partitions	**`separate()`**	split blocks as list
`bdim()`	block structure dim	**`blockdim <-`**	modify block-dimension
`brow()`	# of blocks by row	**`is.bmatrix()`**	test if blockmatrix
`bcol()`	# of blocks by column	**`as.bmatrix()`**	convert to blockmatrix

BLOCK MATRIX MULTIPLICATIONS

CONTRIBUTIONS (2)

- Introduce new notations for matrix products.
- Provide (16) products for block matrices.



MATRIX PRODUCTS

Well Known

$$\underbrace{\mathbf{X}}_{(n_{\mathbf{X}},p_{\mathbf{X}})} = \underbrace{\mathbf{A}}_{(n_{\mathbf{A}},p_{\mathbf{A}})} *_{\omega} \underbrace{\mathbf{B}}_{(n_{\mathbf{B}},p_{\mathbf{B}})} \qquad \omega \in \{s,h,u\}$$

Names	Notations	Validity	Dimension of X	Element Definition
Scalar	$*_s$	$n_{\mathbf{A}} = p_{\mathbf{A}} = 1$	$(n_{\mathbf{X}},p_{\mathbf{X}}) = (n_{\mathbf{B}},p_{\mathbf{B}})$	$x_{ij} = ab_{ij}$
Hadamard	$*_h$	$(n_{\mathbf{A}},p_{\mathbf{A}}) = (n_{\mathbf{B}},p_{\mathbf{B}})$	$(n_{\mathbf{X}},p_{\mathbf{X}}) = (n_{\mathbf{B}},p_{\mathbf{B}})$	$x_{ij} = a_{ij}b_{ij}$
Usual	$*_u$	$p_{\mathbf{A}} = n_{\mathbf{B}}$	$(n_{\mathbf{X}},p_{\mathbf{X}}) = (n_{\mathbf{A}},p_{\mathbf{B}})$	$x_{ij} = \sum_{k=1}^{k=p_{\mathbf{A}}} a_{ik}b_{kj}$

$$\underbrace{\mathbf{A}}_{(2,2)} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\underbrace{\mathbf{B}}_{(2,2)} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$\underbrace{\mathbf{X}}_{(2,2)} = \lambda *_s \mathbf{B} = \begin{bmatrix} 4\lambda & 2\lambda \\ 3\lambda & \lambda \end{bmatrix}$
$\underbrace{\mathbf{X}}_{(2,2)} = \mathbf{A} *_h \mathbf{B} = \begin{bmatrix} 1\times 4 & 2\times 2 \\ 3\times 3 & 4\times 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 6 & 4 \end{bmatrix}$
$\underbrace{\mathbf{X}}_{(2,2)} = \mathbf{A} *_u \mathbf{B} = \begin{bmatrix} 1\times 4+2\times 3 & 1\times 2+2\times 1 \\ 3\times 4+4\times 3 & 3\times 2+4\times 1 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 24 & 10 \end{bmatrix}$

KRONECKER PRODUCT

Well Known

$$\left(\underbrace{\mathbf{X}}_{(n_{\mathbf{X}},p_{\mathbf{X}})} = \underbrace{\mathbf{A}}_{(n_{\mathbf{A}},p_{\mathbf{A}})} \otimes \underbrace{\mathbf{B}}_{(n_{\mathbf{B}},p_{\mathbf{B}})} \right)$$

$$\underbrace{\mathbf{X}}_{(n_{\mathbf{X}},p_{\mathbf{X}})} = \underbrace{\mathbf{A}}_{(n_{\mathbf{A}},p_{\mathbf{A}})} *_k \underbrace{\mathbf{B}}_{(n_{\mathbf{B}},p_{\mathbf{B}})}$$

Names	Notations	Validity	Dimension of X	Element Definition
Kronecker	$*_k$	always	$(n_{\mathbf{X}}, p_{\mathbf{X}}) = (n_{\mathbf{A}} n_{\mathbf{B}}, p_{\mathbf{A}} p_{\mathbf{B}})$	$\mathbf{X}_{ij} = a_{ij}\mathbf{B}$

$$\underbrace{\mathbf{A}}_{(2,2)} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \underbrace{\mathbf{B}}_{(2,2)} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

$$\underbrace{\mathbf{X}}_{(4,4)} = \mathbf{A} *_k \mathbf{B} = \begin{bmatrix} 1\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} & 2\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \\ 3\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} & 4\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 4 & 1 & 8 & 2 \\ 3 & 1 & 6 & 2 \\ 12 & 6 & 16 & 8 \\ 9 & 3 & 12 & 4 \end{bmatrix}$$

BLOCK MATRIX PRODUCTS

$$\underbrace{\mathbf{X}}_{(n_{\mathbf{X}},p_{\mathbf{X}})} = \underbrace{\mathbf{A}}_{(n_{\mathbf{A}},p_{\mathbf{A}})} *_{(\omega_1,\omega_2)} \underbrace{\mathbf{B}}_{(n_{\mathbf{B}},p_{\mathbf{B}})} \qquad \begin{array}{l} \omega_1 \in \{s,h,u,k\} \\ \omega_2 \in \{s,h,u,k\} \end{array}$$

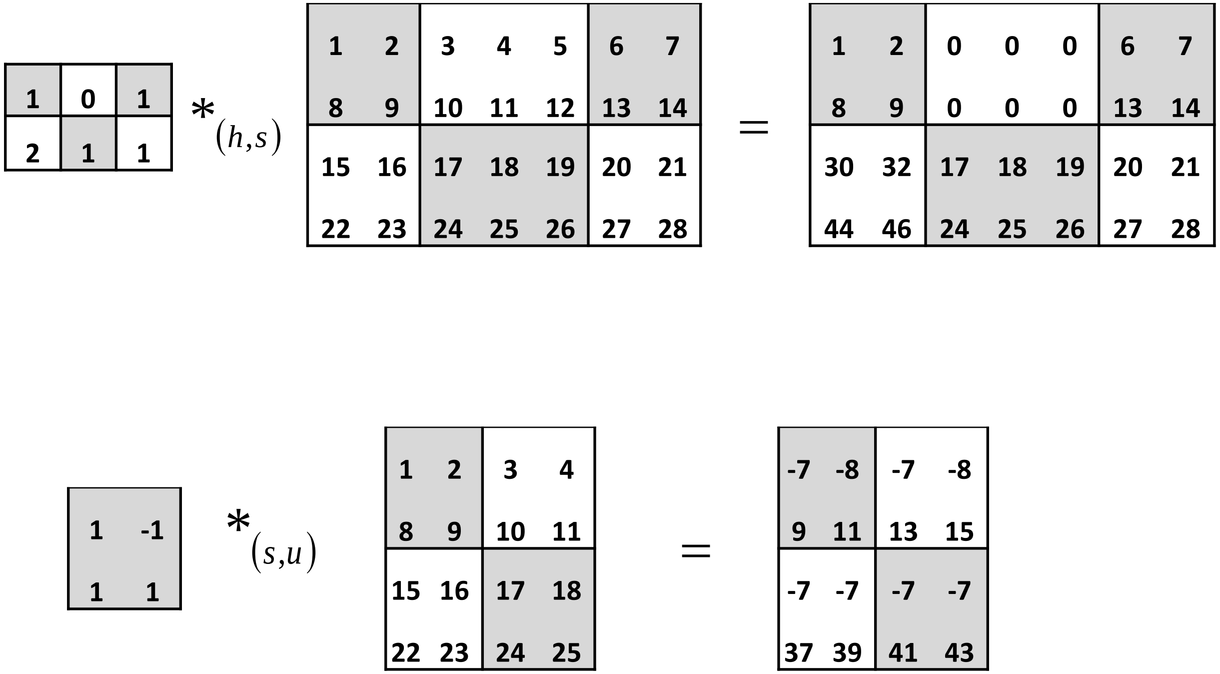
MAIN IDEA → **COMBINE MATRIX PRODUCTS AND PRODUCTS FORMULAS.**

(16) BLOKS MATRIX PRODUCTS = (4) MATRIX PRODUCTS * (4) PRODUCT FORMULAS

$*_{(s,s)}$	Known
$*_{(s,h)}$	New
$*_{(s,u)}$	New
$*_{(h,s)}$	New
$*_{(h,h)}$	New
$*_{(h,u)}$	New
$*_{(u,s)}$	New
$*_{(u,h)}$	Known
$*_{(u,u)}$	Known

$*_{(s,k)}$	Known
$*_{(h,k)}$	New
$*_{(u,k)}$	New
$*_{(k,s)}$	New
$*_{(k,h)}$	New
$*_{(k,u)}$	New

EXAMPLE



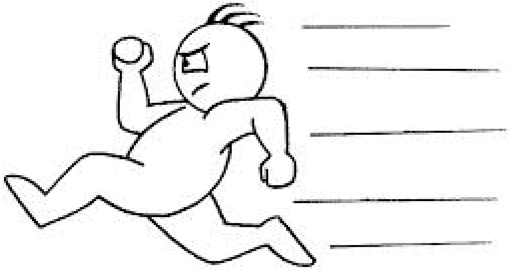
$$\begin{array}{|c|c|c|}\hline 1 & 0 & 1 \\ \hline 2 & 1 & 1 \\ \hline \end{array} *_{(h,s)} \left[\begin{array}{cc|ccc|cc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ \hline 15 & 16 & 17 & 18 & 19 & 20 & 21 \\ 22 & 23 & 24 & 25 & 26 & 27 & 28 \end{array}\right] = \left[\begin{array}{cc|ccc|cc} 1 & 2 & 0 & 0 & 0 & 6 & 7 \\ 8 & 9 & 0 & 0 & 0 & 13 & 14 \\ \hline 30 & 32 & 17 & 18 & 19 & 20 & 21 \\ 44 & 46 & 24 & 25 & 26 & 27 & 28 \end{array}\right]$$

$$\left[\begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array}\right] *_{(s,u)} \left[\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 8 & 9 & 10 & 11 \\ \hline 15 & 16 & 17 & 18 \\ 22 & 23 & 24 & 25 \end{array}\right] = \left[\begin{array}{cc|cc} -7 & -8 & -7 & -8 \\ 9 & 11 & 13 & 15 \\ \hline -7 & -7 & -7 & -7 \\ 37 & 39 & 41 & 43 \end{array}\right]$$

Block Matrix products in BlockBerry

Notations	R operator	Remark
$*_{(s,s)}$	%ss%	✓
$*_{(s,h)}$	%sh%	✓
$*_{(s,u)}$	%su%	✓
$*_{(h,s)}$	%hs%	✓
$*_{(h,h)}$	%hh%	✓
$*_{(h,u)}$	%hu%	✓
$*_{(u,s)}$	%us%	In progress
$*_{(u,h)}$	%uh%	✓
$*_{(u,u)}$	%uu%	✓

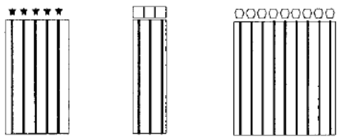
Notations	R operator	Remark
$*_{(k,s)}$	%ss%	In progress
$*_{(k,h)}$	%sh%	
$*_{(k,u)}$	%su%	
$*_{(s,k)}$	%hs%	
$*_{(h,k)}$	%hh%	
$*_{(u,k)}$	%hu%	



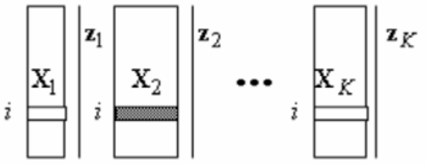
FAST PROTOTYPING BLOCK ALGORITHMS

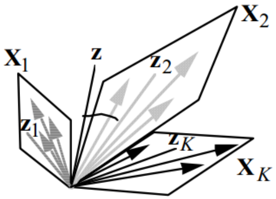
MULTIBLOCK METHODS

Examples of multiblock methods based on canonical variate principle



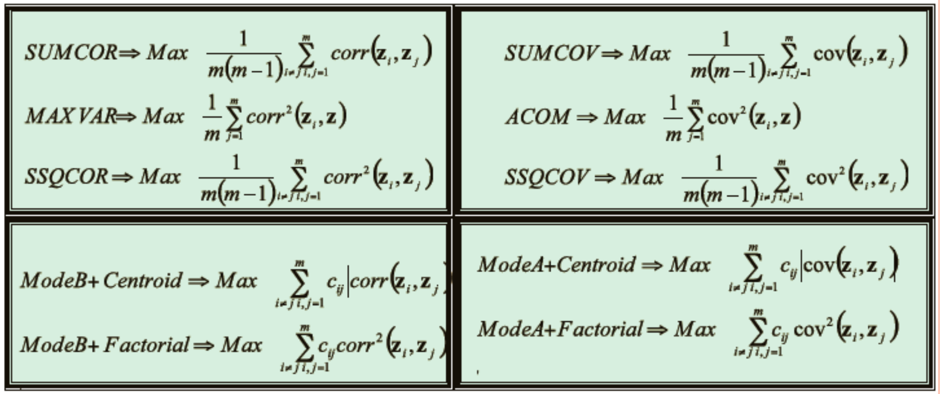
$\mathbf{z}_1 = \mathbf{X}_1\mathbf{a}_1, \mathbf{z}_2 = \mathbf{X}_2\mathbf{a}_2, \dots, \mathbf{z}_K = \mathbf{X}_K\mathbf{a}_K$



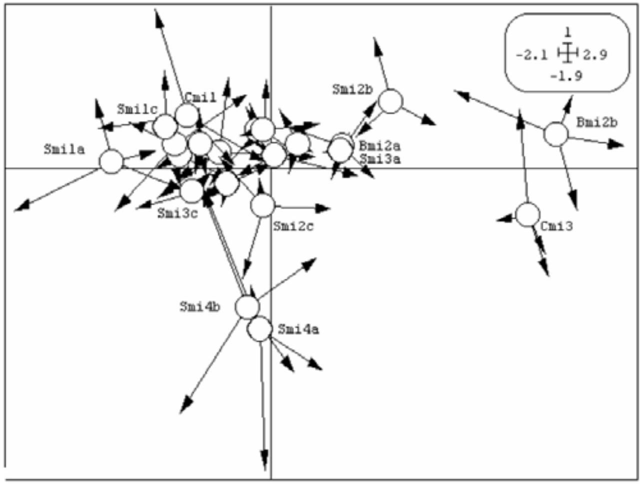


Example with Maxbet method

$$\max_{\|\mathbf{a}_k\|=1} \sum_{k,l=1}^{K} \operatorname{cov}(\mathbf{z}_k, \mathbf{z}_l) \Leftrightarrow \max_{\|\mathbf{a}_k\|=\|\mathbf{z}\|=1} \sum_{k=1}^{K} \operatorname{cov}(\mathbf{z}_k, \mathbf{z})$$

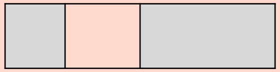
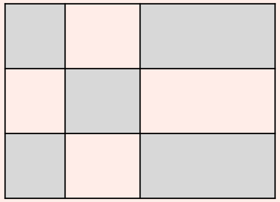


$SUMCOR \Rightarrow Max \ \frac{1}{m(m-1)} \sum_{i\neq j\, i,j=1}^{m} corr(\mathbf{z}_i, \mathbf{z}_j)$ $MAXVAR \Rightarrow Max \ \frac{1}{m} \sum_{j=1}^{m} corr^2(\mathbf{z}_i, \mathbf{z})$ $SSQCOR \Rightarrow Max \ \frac{1}{m(m-1)} \sum_{i\neq j\, i,j=1}^{m} corr^2(\mathbf{z}_i, \mathbf{z}_j)$	$SUMCOV \Rightarrow Max \ \frac{1}{m(m-1)} \sum_{i\neq j\, i,j=1}^{m} \operatorname{cov}(\mathbf{z}_i, \mathbf{z}_j)$ $ACOM \Rightarrow Max \ \frac{1}{m} \sum_{j=1}^{m} \operatorname{cov}^2(\mathbf{z}_i, \mathbf{z})$ $SSQCOV \Rightarrow Max \ \frac{1}{m(m-1)} \sum_{i\neq j\, i,j=1}^{m} \operatorname{cov}^2(\mathbf{z}_i, \mathbf{z}_j)$
$ModeB+Centroid \Rightarrow Max \ \sum_{i\neq j\, i,j=1}^{m} c_{ij} \lvert corr(\mathbf{z}_i, \mathbf{z}_j$ $ModeB+Factorial \Rightarrow Max \ \sum_{i\neq j\, i,j=1}^{m} c_{ij} corr^2(\mathbf{z}_i, \mathbf{z}_j)$	$ModeA+Centroid \Rightarrow Max \ \sum_{i\neq j\, i,j=1}^{m} c_{ij} \lvert \operatorname{cov}(\mathbf{z}_i, \mathbf{z}_j) \rvert$ $ModeA+Factorial \Rightarrow Max \ \sum_{i\neq j\, i,j=1}^{m} c_{ij} \operatorname{cov}^2(\mathbf{z}_i, \mathbf{z}_j)$



MAXBET ALGORITHMS

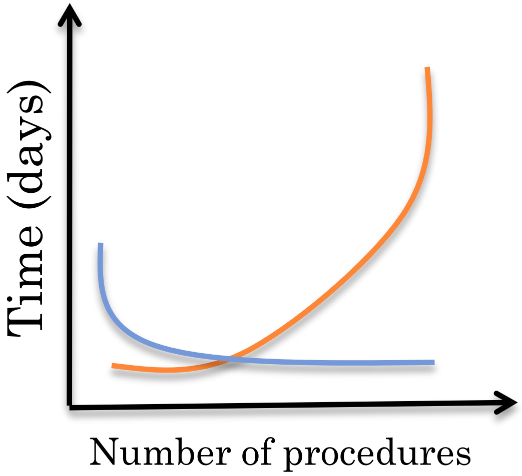
$$\max_{\|\mathbf{a}_k\|=1} \sum_{k,l=1}^{K} \operatorname{cov}(\mathbf{z}_k, \mathbf{z}_l) \Leftrightarrow \max_{\|\mathbf{a}_k\|=\|\mathbf{z}\|=1} \sum_{k=1}^{K} \operatorname{cov}(\mathbf{z}_k, \mathbf{z})$$

Cores	Representation	Iteration
$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_K \end{bmatrix}$		$\begin{cases} \mathbf{z}^{(s+1)} = \dfrac{\sum_{l=1}^{K} \mathbf{X}_k \mathbf{a}_k^{(s)}}{\left\| \sum_{l=1}^{K} \mathbf{X}_k \mathbf{a}_k^{(s)} \right\|} \\ \mathbf{a}_k^{(s+1)} = \dfrac{\mathbf{X}_k' \mathbf{z}^{(s+1)}}{\left\| \mathbf{X}_k' \mathbf{z}^{(s+1)} \right\|} \end{cases}$
$\mathbf{A} = \mathbf{X}'\mathbf{X} = \begin{bmatrix} \mathbf{X}_1'\mathbf{X}_1 & \mathbf{X}_1'\mathbf{X}_2 & \cdots & \mathbf{X}_1'\mathbf{X}_K \\ \mathbf{X}_2'\mathbf{X}_1 & \mathbf{X}_2'\mathbf{X}_2 & \cdots & \mathbf{X}_2'\mathbf{X}_K \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_K'\mathbf{X}_1 & \mathbf{X}_K'\mathbf{X}_2 & \cdots & \mathbf{X}_K'\mathbf{X}_K \end{bmatrix}$		$\mathbf{a}_k^{(s+1)} = \dfrac{\sum_{l=1}^{K} \mathbf{X}_l' \mathbf{X}_k \mathbf{a}_k^{(s)}}{\left\| \sum_{l=1}^{K} \mathbf{X}_l' \mathbf{X}_k \mathbf{a}_k^{(s)} \right\|} \left(1 \le k \le K\right)$

MAXBET ALGORITHMS

Matrix notations	Block matrix notations
$\begin{cases} \mathbf{z}^{(s+1)} = \dfrac{\sum_{l=1}^{K} \mathbf{X}_k \mathbf{a}_k^{(s)}}{\left\lVert \sum_{l=1}^{K} \mathbf{X}_k \mathbf{a}_k^{(s)} \right\rVert} \\ \mathbf{a}_k^{(s+1)} = \dfrac{\mathbf{X}_k^{'} \mathbf{z}^{(s+1)}}{\left\lVert \mathbf{X}_k^{'} \mathbf{z}^{(s+1)} \right\rVert} \end{cases}$	$\begin{cases} \mathbf{z}^{(s+1)} = \dfrac{\mathbf{X} *_u \mathbf{a}}{\left\lVert \mathbf{X} *_u \mathbf{a} \right\rVert} \\ \mathbf{a}^{(s+1)} = \dfrac{\mathbf{X}^{'} *_{(h,u)} \mathbf{z}^{(s+1)}}{\left\lVert \mathbf{X}^{'} *_{(h,u)} \mathbf{z}^{(s+1)} \right\rVert} \end{cases}$
$\mathbf{a}_k^{(s+1)} = \dfrac{\sum_{l=1}^{K} \mathbf{X}_l^{'} \mathbf{X}_k \mathbf{a}_k^{(s)}}{\left\lVert \sum_{l=1}^{K} \mathbf{X}_l^{'} \mathbf{X}_k \mathbf{a}_k^{(s)} \right\rVert} \left(1 \le k \le K\right)$	$\mathbf{a}^{(s+1)} = \dfrac{\left(\mathbf{X}^{'} *_{(u,u)} \mathbf{X}\right) *_{(h,u)} \mathbf{a}^{(s)}}{\left\lVert \left(\mathbf{X}^{'} *_{(u,u)} \mathbf{X}\right) *_{(h,u)} \mathbf{a}^{(s)} \right\rVert} \left(1 \le k \le K\right)$

MAIN CONCLUSIONS.



- An algebraic framework that makes feasible the translation of conceptual knowledge into software tools
- The large number of block matrix products is explained by multiple views to work with block matrices. (not all products presented here)
- The proposed architecture can be implemented in other scientific software (Matlab,…)

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